A note on Lorentz-covariant theories of gravitation

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A note on Lorentz-covariant theories of gravitation

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Abstract. The most recent Lorentz-covariant theory of gravitation is considered. It is shown how one could easily deduce this theory from Einstein's general relativity, so that agreement with the three celebrated tests follows as a trivial consequence, without need for calculations.

The recent article by Coleman (1971) lends itelf to several considerations and interesting comments. In my opinion it is important to note that, contrary to one's expectation on the basis of the author's observations, this work is actually part of Einstein's general relativity. It is well known, in fact, that the coupling of the variational principle

$$\delta \int \mathrm{d}\sigma = 0 \tag{1}$$

with the invariant interval definition

$$\mathrm{d}\sigma^2 = g_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k \tag{2}$$

necessarily leads to the celebrated equations of motion of a test particle in an external gravitational field

$$\frac{\mathrm{d}^2x^i}{\mathrm{d}\sigma^2} + \Gamma_{kl}{}^i \frac{\mathrm{d}x^k}{\mathrm{d}\sigma}\frac{\mathrm{d}x^l}{\mathrm{d}\sigma} = 0 \tag{3}$$

where $\Gamma_{kl}{}^i$ is the usual Christoffel symbol.

From this point of view, Coleman's theory cannot be considered as an alternative to Einstein's, for it actually assumes the geodesics in curved space–time

$$\mathrm{D}u^i = 0 \tag{4}$$

to be the world lines of massive objects (D denotes covariant differentiation and u^i is the four velocity). This is just the central idea of Einstein's theory.

According to Coleman, the simplicity of his theory must be traced back to the choice of a simple metric tensor or equivalently to the invariant quadratic form

$$\mathrm{d}\sigma^2 = \mathrm{e}^{2\phi}\,\mathrm{d}t^2 - c^{-2}\,\mathrm{e}^{-2\phi}\,(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2) \tag{5}$$

and I think that the problem may be profitably discussed here in some detail.

In my opinion the use of the metric (5) does not introduce any meaningful simplification into the theory. It is only the choice of a particular metric among the wide variety of those admissible in Einstein's theory, where the invariant interval for every field must reduce to the form

$$d\sigma^2 = (1+2\varphi)\,dt^2 - \frac{1}{c^2}(1-2\varphi)(dx^2+dy^2+dz^2) \tag{6}$$

in the static and weak field approximation (Møller 1952, p 316 formula (37)). This is the compelling metric for all static fields created by massive bodies at a distance so large that the field may be considered weak, and at the same time so small as to be negligible with respect to the length of gravitational waves typical of the system (Landau and Lifshitz 1959). From (6), through the simple substitution

$$1 \pm 2\varphi \rightarrow e^{\pm 2\varphi} \tag{7}$$

one obtains exactly Coleman's metric (5) which will obviously reproduce in the first order approximation the same results as general relativity. It should be clear from above that this is essentially an involutive process, equivalent to the chain

$$1 \pm 2\varphi \rightarrow e^{\pm 2\varphi} \simeq 1 \pm 2\varphi \tag{8}$$

where the intermediate step may be lifted to the dignity of Lorentz-invariant scalar theory of gravitation. It should be apparent from the foregoing that so long as we do not take into account higher order effects, such as the probem of gravitational wave radiation, nobody will ever be able to tell which theory is correct. Luckily enough, Einstein's theory was not merely devised to stand the three gravitational tests. It means much more than that, its cosmological implications being one example.

Situations resembling that investigated in the present note are possible only because the verification of gravitational theories rests on such a limited number of effects that it is hardly possible to separate Einstein's work out of the cluster of its 'rivals' which are being relentlessly produced every year. All of them claim to agree with the three celebrated tests: perihelion advance, gravitational red shift and photon deflection. Moreover, since all calculations (Coleman's work is an example) are invariably carried out in the weak field approximation, virtually every physicist in the world may be able to exhibit his own gravitational theory.

References

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